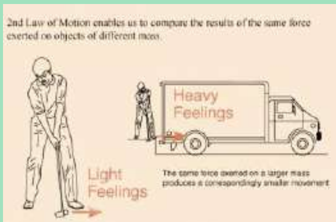
- Imagine a mass is to be moved **linearly** in a certain direction.
- According to Newton's 1st and 2nd Law of Motion ($F = ma$),
  1. A **force** must be added to **change its speed** or to **create a linear acceleration.**
  2. **Greater the mass** of the body, **greater the force** is needed to create **same linear acceleration**.



What if we want to move a body <u>angularly</u> instead of <u>linearly</u>???

Does <u>mass</u> the only culprit to hinder the rotation of a body???

Let us suppose a ball/ hammer throwing game.

<u>Not only mass</u> here is responsible to resist the rotational motion of a body, but also <u>distance of the body from the rotation axis</u> is involved here.

Nik Wallenda crossing Niagara Falls over a tightrope in 2012 and became the first to walk on a rope over Niagara Falls.

He used a long rod to balance himself and distribute the total weight over a wide distance thus **increasing moment of inertia that is resistance of rotation.**

ME 141
Engineering Mechanics

Portion 7
Inertia

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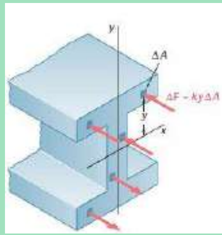
Moment of Inertia of an Area (Second Moment of Area)

$R = \int ky\, dA = k \int y\, dA$

$M = \int ky \times y\, dA = k \int y^2\, dA$

The last integral is known as the **second moment of area**, or **moment of inertia of the area.**

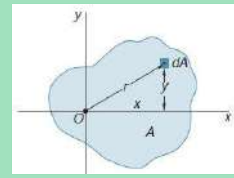


Polar Moment of Inertia

$Jo = \int r^2\, dA$

Jo is the **polar moment of inertia with respect to the pole O.**

It is useful in case of cylindrical or spherical body in rotation.



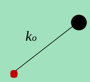
$Jo = \int r^2\, dA = \int (x^2 + y^2)\, dA = \int x^2\, dA + \int y^2\, dA = Ix + Iy$

Radius of Gyration

Rotation of **any body** like gear rotation can be converted to the **rotation of a point of same mass of the body but at a certain distance from the rotation axis**.

This distance is known as **Radius of Gyration (k_o)**.

$I_o = \int r^2\, dA$



$I_o = k_o^2 A = \int r^2\, dA$

So, radius of gyration with respect to O,

$k_o = \sqrt{(I_o / A)}$

Similarly, radius of gyration with respect *x*-axis

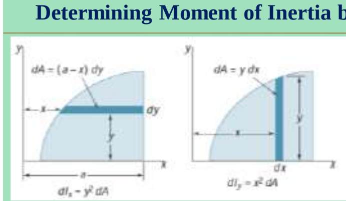
$kx = \sqrt{(Ix / A)}$

Radius of gyration with respect *y*-axis

$ky = \sqrt{(Iy / A)}$

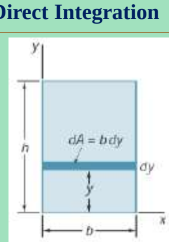
And, $k_o^2 = kx^2 + ky^2$

Determining Moment of Inertia by Direct Integration

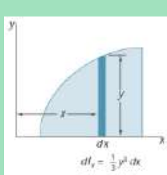


$Ix = \int y^2\, dA = \int dIx$

$Iy = \int x^2\, dA = \int dIy$

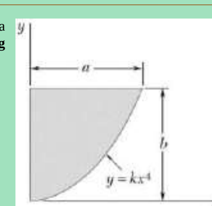


$Ix = \int y^2\, dA = \int dIx = 1/3\, bh^3$



Problem 7.1 (Beer Johnston_10th edition_P9.4 and P9.8)

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* and *y* axis **using strips parallel to *x* and *y*-axis.**

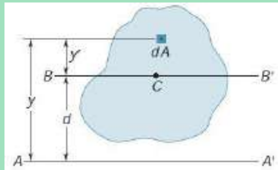


PARALLEL-AXIS THEOREM

Consider the moment of inertia I of an area A with respect to an axis AA' .

$I = \int y^2 \, dA$

Let us now draw a centroidal axis BB' through the centroid C of the area parallel to AA'.



$I = \int y^2 \, dA = \int (y'+d)^2 \, dA$

$= \int y'^2 \, dA + 2d \int y' \, dA + d^2 \int dA$

The first integral represents the moment of inertia I of the area with respect to the centroidal axis BB'. The second integral represents the first moment of the area with respect to BB'; since the centroid C of the area is located on that axis, the second integral must be zero.
Therefore,

$$I = \bar{I} + A\, d^2$$

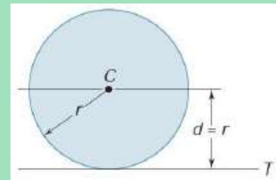
Also for radius of gyration,

$$k^2 = \bar{k}^2 + d^2$$

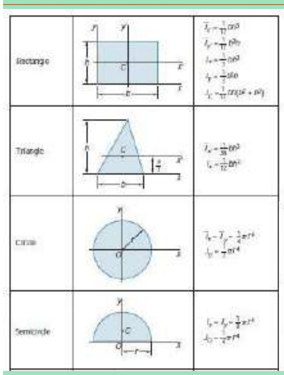
➢For more detail see **Vector Mechanics for Engineers: Statics and Dynamics-** Ferdinand Beer, Jr., E. Russell Johnston, David Mazurek, Phillip Cornwell.

Class Performance

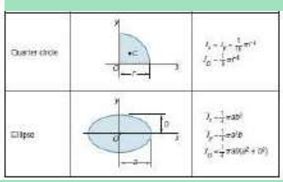
Find the moment of inertia of the circular area *A*, about the axis *T*. Knowing that moment of inertia of the of the area about the centroidal axis is (1/4) πr^4



MOMENTS OF INERTIA OF COMMON AREAS

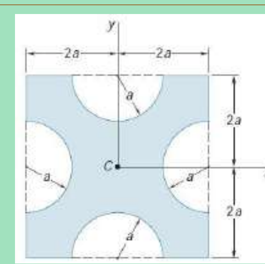


For measuring the moment of inertia of composite areas, it may be needed to transfer the moment of inertia about desired axis using parallel axis theorem and then arithmetically added them all.



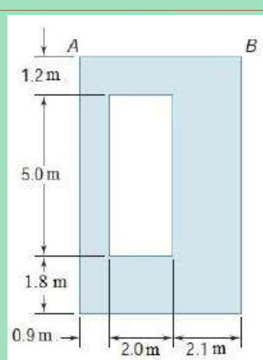
Problem 7.2 (Beer Johnston_10th edition_P9.35)

Determine the moments of inertia of the shaded area shown with respect to the x and y axes when a = 20 mm.

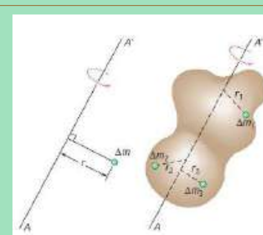


Problem 7.3 (Beer Johnston_10th edition_P9.43)

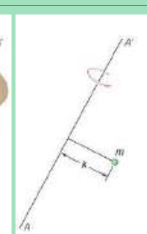
Determine the moments of inertia *Ix* and *Iy* of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.



MOMENT OF INERTIA OF A MASS



$I = \int r^2 \, dm$



$I = k^2 m$

End of Portion 7

References

➢**Vector Mechanics for Engineers: Statics and Dynamics**
Ferdinand Beer, Jr., E. Russell Johnston, David Mazurek, Phillip Cornwell.